Automatic Control (1)

By

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Lecture (1)

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Course Title: Automatic Control (1)

Course Code: EEC 321

Prerequisites: Signals and Systems

Study Hours: 3 Cr. hrs.

= [2 Lect. + 2 Tut + 0 Lab]

Assessment:

Final Exam: 40%.

Midterm: 1ˢᵗ 20% & 2ⁿᵈ 20%.

Year Work & Quizzes: 15%.

MATLAB Mini Project: 5%.

Textbook:

1. Nise, N. S. "Control System Engineering", 7th edition, John Wiley & Sons Ltd., UK, 2016.
2. Katsuhiko Ogata, "Modern Control Engineering", Prentice Hall, 5th Edition.
3. F. Golnaraghi and B. C. Kuo, "Automatic control Systems", 10th ed., John Wiley & Sons, Inc. 2017.
4. Andrea Bacciotti, "Stability and Control of Linear Systems" Volume 185, Springer, 2019

Course Description

➢ Introduction to control systems: Open loop systems, closed loop systems, transfer function and concept of poles and zeros. Block diagram reduction techniques. Signal flow graph techniques. Modeling of some electrical, mechanical and thermal systems. Time response of first and second order systems, PID controllers. Steady-state error. Concept of stability: Routh's stability criterion.

➢ Root locus method. State- space representation: State model of linear systems using physical variables, State, space representation using phase variables. Properties of transition matrix and solution of state equation. compensation. Programable

Analysis & Design
# of Control Systems using Root Locus

Introduction

Motivation:

To satisfy transient performance requirements, it may be necessary to know how to choose certain controller parameters so that the resulting closed-loop poles are in the performance regions, which can be solved with Root Locus technique.

Definition:

A graph displaying the roots of a polynomial equation when one of the parameters in the coefficients of the equation changes from 0 to ∞.
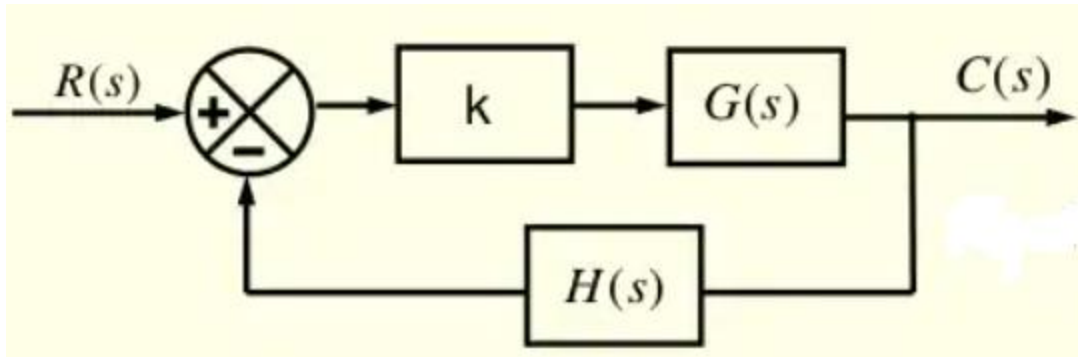
- Consider the feedback control system shown in figure, the closed loop transfer function is given by:

$$T(s) = \frac{C(s)}{R(s)} = \frac{k\,G(s)}{1 + k\,G(s)H(s)}$$

- Where k is a constant gain parameter. The poles of the transfer function are the roots of the characteristic equation given by:

$$1 + k\,G(s)\,H(s) = 0$$

Definition of Root Locus

- The root locus is the locus of the characteristic equation of the closed- loop system as a specific parameter (usually, gain K) is varied from zero to infinity. For $0 \leq k < \infty$, if $\rightarrow$ $1 + k\,G(s)\,H(s) = 0$

- Then $G(s)H(s) = -\dfrac{1}{k}$

- and for *positive k*, this means that a *point s* which is a point on locus must satisfy the magnitude and angle conditions:

➢ The magnitude condition $\rightarrow$ $|\,G(s)H(s)\,| = \dfrac{1}{k}$

➢ The angle condition $\rightarrow$ $\angle G(s)H(s) = 180^{o} \pm i360^{o}\ where\quad i = 0, \pm 1, \pm 2, \pm 3, ...$

Root Locus Rules

- **_Rule 1:_**

The number of branches of the root locus is equal to the number of closed-loop poles (or roots of the characteristic equation).

- **_Rule 2:_**

Root locus starts at open-loop poles (when K= 0) and ends at open-loop zeros (when K=∞). If the number of open-loop poles is greater than the number of open-loop zeros, some branches starting from finite open-loop poles will terminate at zeros at infinity.

- ***Rule 3:***

Root locus is symmetric about the real axis, which reflects the fact that closed-loop poles appear in complex conjugate pairs.

- ***Rule 4:***

Along the real axis, the root locus includes all segments that are to the left of an <u>odd</u> number of finite real open-loop poles and zeros.

- ***Rule 5:***

If number of poles (n) exceeds the number of zeros (m), then as K $\longrightarrow \infty$, (n - m) branches will become asymptotic to straight lines. These straight lines intersect the real axis with angles θ_A at σ_A.

$$\sigma_A = \frac{\sum p_i - \sum z_i}{n - m} = \frac{sum\,open\,loop\,poles\ -\ sum\,open\,loop\,zeros}{\#\,of\,open\,loop\,poles - \#\,of\,open\,loop\,zeros}$$

$$\theta_A = \frac{180 + i360}{n - m} \qquad where \quad i = 0, \pm1, \pm2, \pm3, ...$$

- ***Rule 6:***

"Breakaway and/or break-in points" where the locus between two poles on the real axis leaves the real axis is called the breakaway point and the point where the locus between two zeros on the real axis returns to the real axis is called the break-in point. The loci leave or return to the real axis at the maximum *gain k* of the following equation:

$$|G(s_o)H(s_o)| = \frac{1}{k}$$

- ***Rule 7:***

The departure angle for a *pole pi* (the arrival angle for a zero zi) can be calculated by slightly modifying the following equation:

➢ ***The angle of departure φ_p of a locus from a complex pole is given by:***

φ_p = 180 − [sum of the other GH pole angles to the pole under consideration] + [sum of the GH zero angles to the pole]

The angle of approach φ_z of a locus to a complex zero is given by:

φ_z = 180 + [sum of the other GH pole angles to the zero under consideration] − [sum of the GH zero angles to the zero]

- ***<u>Rule 8:</u>***

If the root locus passes through the imaginary axis (the stability boundary), the crossing point jw and the corresponding *<u>gain K</u>* can be found using Routh-Hurwitz criterion.

Steps to Sketch Root Locus

- **_Step 1 :_**

Transform the closed-loop characteristic equation into the standard form for sketching root locus:

$$1 + k\,G(s)\,H(s) = 0$$

- **_Step2:_**
- ➢ Find the open-loop zeros and the open-loop poles.
- ➢ Mark the open-loop poles and zeros on the complex plane.
- ➢ Use '_x_' to represent open-loop poles and '_o_' to represent the open-loop zeros.

- ***Step3:***

Determine the real axis segments that are on the root locus by applying **Rule 4**.

- ***Step4:***

(If necessary) Determine the number of asymptotes and the corresponding intersection and angles by applying **Rules 2 and 5**.

- ***Step5:***

(If necessary) Determine the break-away and break-in points using **Rule 6.**

- **_Step 6:_**

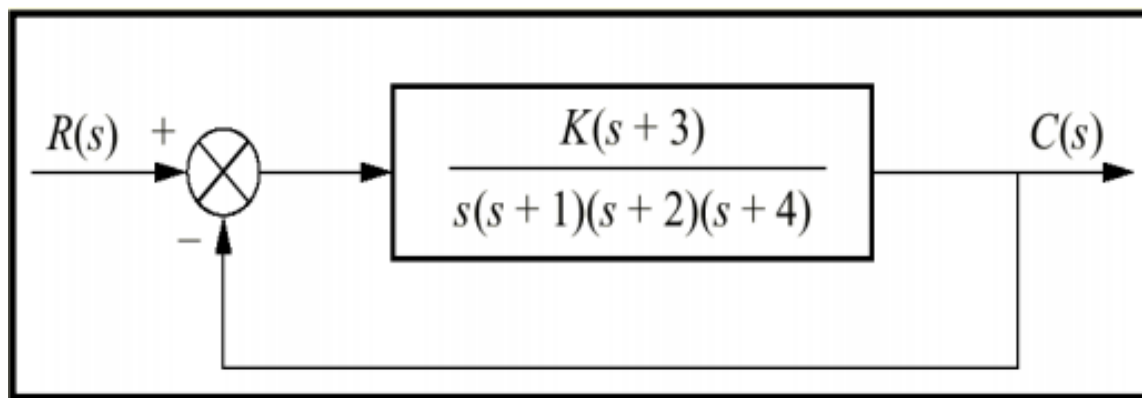(*If necessary*) Determine the departure and arrival angles using **Rule 7**.

- **_Step 7:_**

(*If necessary*) Determine the imaginary axis crossings using **Rule 8.**

- **_Step8:_**

Use the information from Steps 1-7, sketch the root locus.

Root Locus Procedure – Example 1

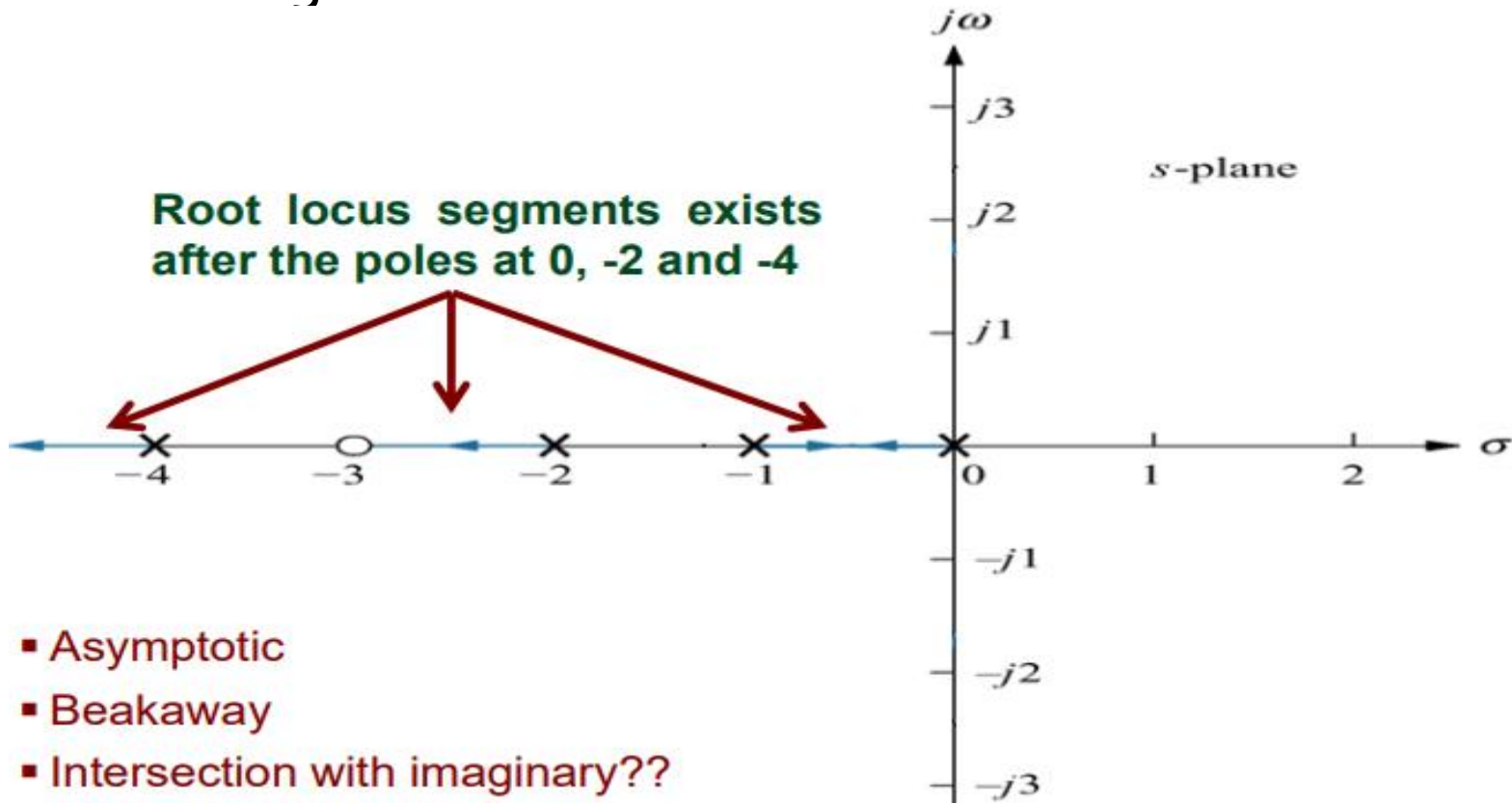**_Example 1:_** Sketch the root locus for the system shown in Figure.

Solution: **_Step 1:_** "Write the open loop transfer function of the system and find the open loop Poles & zeros" as follows:

$$kG(s)H(s) = \frac{k(s+3)}{s(s+1)(s+2)(s+4)}$$

- Open loop Poles = {0, -1, -2, -4} → # of poles = n = 4
- Open loop Zeros = {-3} → # of zeros = m = 1

**Step 2:** "Draw the pole-zero plot of kG(s)H(s), then locate the segments of real axis that are root loci".

Root locus segments exists after the poles at 0, -2 and -4

- Asymptotic
- Beakaway
- Intersection with imaginary??

Step 3: Asymptotic Angles: "As k approaches **+∞**, the branches of the locus become asymptotic to straight lines with angles:

$$\theta_A = \frac{180 + i360}{n - m} \; where \quad i = 0, \pm 1, \pm 2, \pm 3, \ldots.$$

- The number of asymptotes are three [n-m = 3].
- for i=0→ θ1 = 60, for i=1→ θ2=180, and for i= -1→θ3=-60.

Step 4: Center of Asymptotes: The starting point of the asymptotes. These linear asymptotes are centered at a point on the real axis at:

$$\sigma_A = \frac{\sum pole\ values\ of\ GH - \sum zero\ values\ of\ GH}{n - m}$$

$$= \frac{-1 - 2 - 4 + 3}{3} = -1.33$$
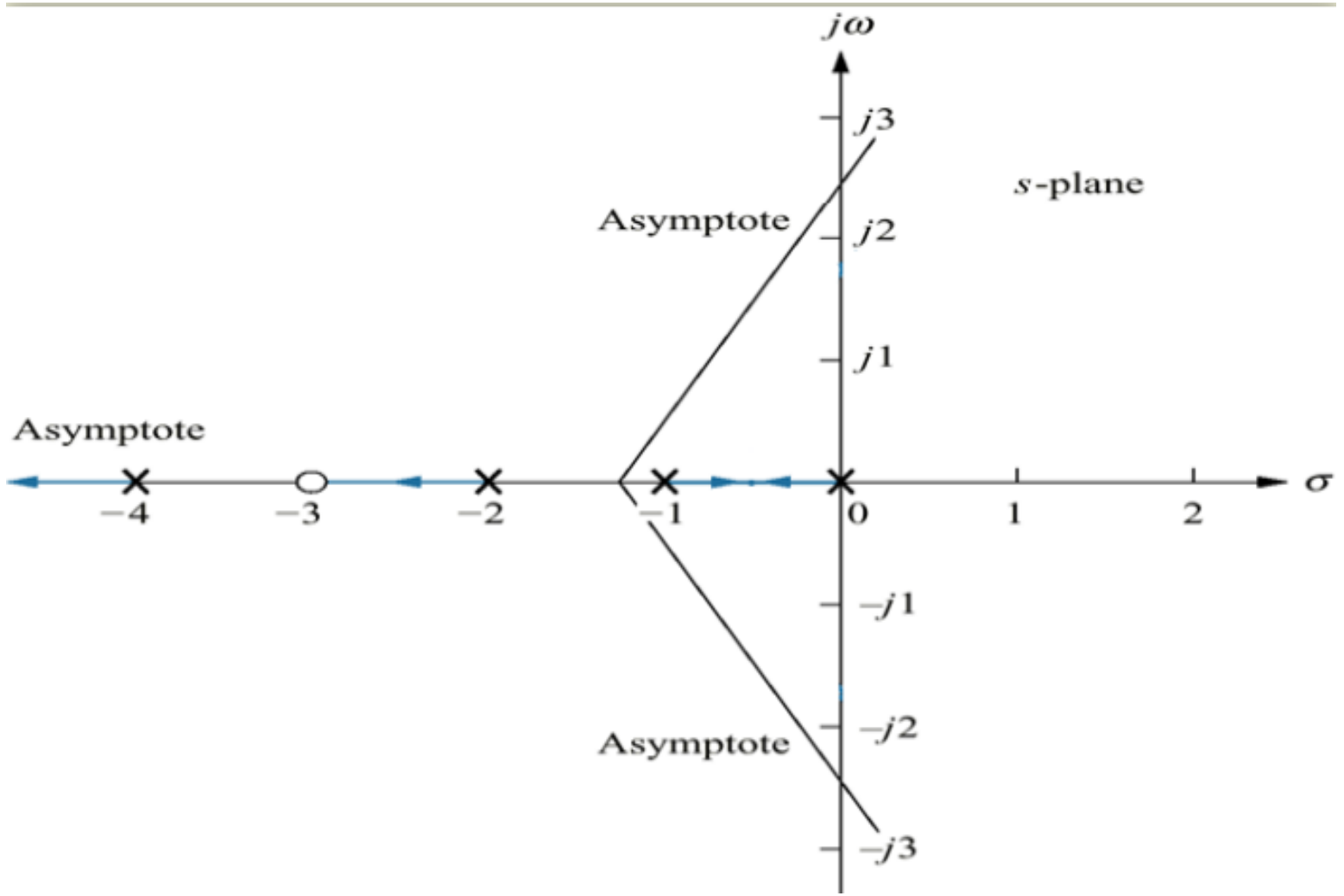
Step 5: Breakaway and break-in or entry points:

$$|G(s_o)H(s_o)| = \frac{1}{k} \quad and \quad k = \frac{1}{|G(s)H(s)|} = \left| \frac{s(s+1)(s+2)(s+4)}{(s+3)} \right|$$

- Therefore, the break away point lies between 0 and -1 at maximum value for **_k_** which is calculated as follows:

S	-0.3	-0.4	-0.45	-0.5
k	0.489	0.532	0.534	0.525

- It is clear that the breakaway point is at s = -0.45.

- Step-4: Determine the *breakaway point* or *break-in point*.

$$G(s)H(s) = \frac{K}{s(s+1)(s+2)}$$

- The characteristic equation of the system is

$$1 + G(s)H(s) = 1 + \frac{K}{s(s+1)(s+2)} = 0$$

$$\frac{K}{s(s+1)(s+2)} = -1$$

$$K = -\left[s(s+1)(s+2)\right]$$

- The breakaway point can now be determined as

$$\frac{dK}{ds} = -\frac{d}{ds}\left[s(s+1)(s+2)\right]$$

- Step-4: Determine the *breakaway point* or *break-in point*.

$$\frac{dK}{ds} = -\frac{d}{ds}\left[s(s+1)(s+2)\right]$$

$$\frac{dK}{ds} = -\frac{d}{ds}\left[s^3 + 3s^2 + 2s\right]$$

$$\frac{dK}{ds} = -3s^2 - 6s - 2$$

- Set *dK/ds=0* in order to determine breakaway point.

$$-3s^2 - 6s - 2 = 0$$

$$3s^2 + 6s + 2 = 0$$

$$s = -0.4226$$

$$= -1.5774$$

- **Step-4**: Determine the *breakaway point* or *break-in point*.

$$s = -0.4226$$
$$= -1.5774$$

$$G(s)H(s) = \frac{K}{s(s+1)(s+2)}$$

- Since the breakaway point must lie on a root locus between 0 and −1, it is clear that *s=−0.4226* corresponds to the actual breakaway point.

- Point s=−1.5774 is not on the root locus. Hence, this point is not an actual breakaway or break-in point.

- In fact, evaluation of the values of K corresponding to s=−0.4226 and s=−1.5774 yields

$$K = 0.3849, \quad \text{for } s = -0.4226$$
$$K = -0.3849, \quad \text{for } s = -1.5774$$

Step 7: jw axis crossing:

- the closed loop transfer function is given by:

$$T(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{k(s+3)}{s^4 + 7s^3 + 14s^2 + (8+k)s + 3k}$$

- The c/s equation is $1 + G(s)H(s) = s^4 + 7s^3 + 14s^2 + (8+k)s + 3k$
- The Routh-table for the characteristic equation is shown.
- The coefficient of $s^1 = 0$

$$-k^2 - 65k + 720 = 0$$
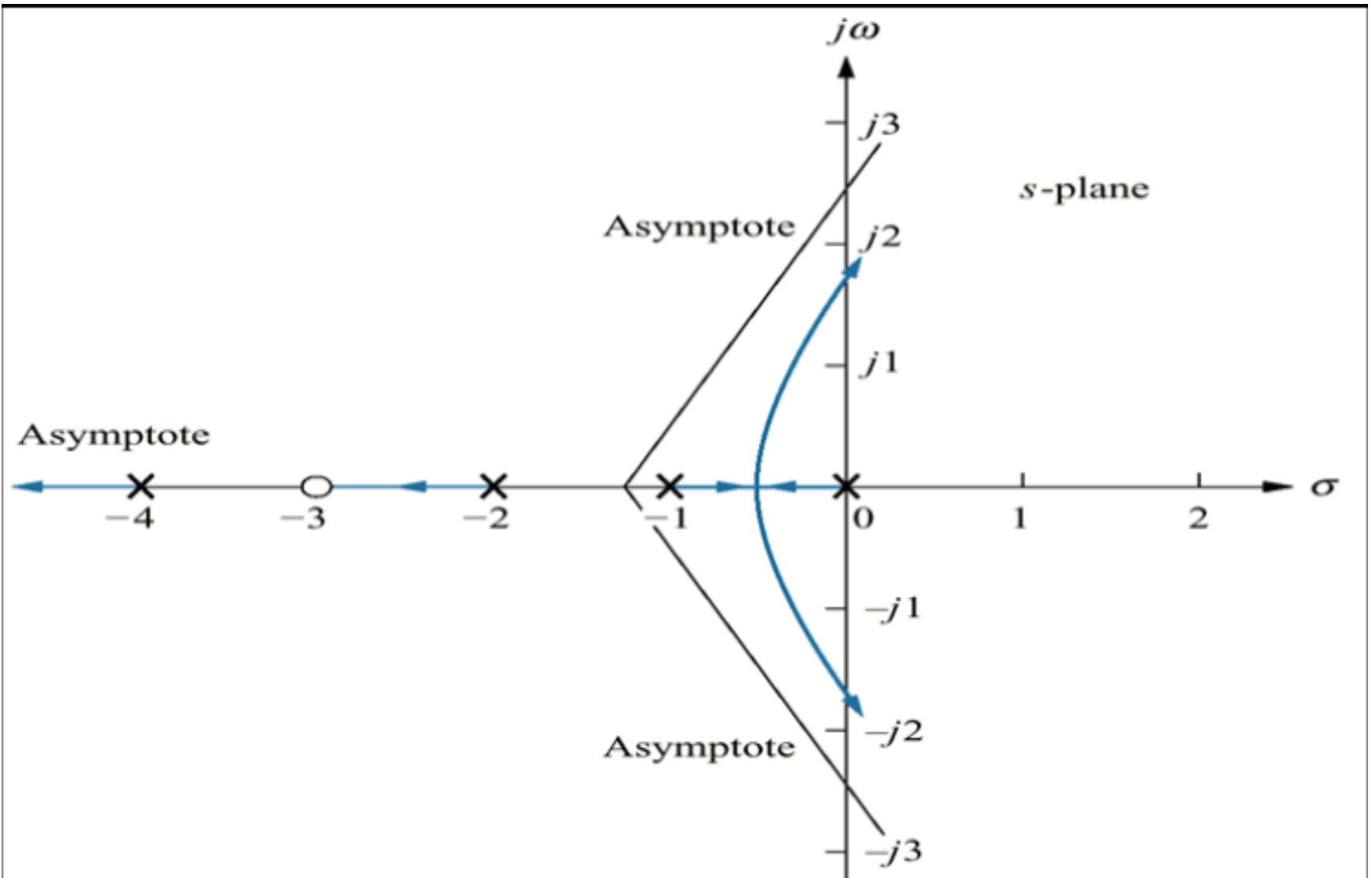
- *Then k = 9.65*
- The intersection points are

$$(90 - k)s^2 + 21k = 0$$

$$80.35s^2 + 202.7 = 0$$

$$thus \ s_{1,2} = \pm j1.59$$

s^4	1	14	$3K$
s^3	7	$8+K$	
s^2	$90 - K$	$21K$	
s^1	$\dfrac{-K^2 - 65K + 720}{90 - K}$		
s^0	$21K$		

We also conclude that the system is stable for 0 < k < 9.65.

Examples

Sketch the root locus for the system with the characteristic equation of;

$$G(s)H(s) = \frac{K}{(s-1)(s+2)(s+3)}$$

1. Number of poles $= n = 3$.
2. Number of zeros $= m = 0$.
3. Number of asymptotes $= n - m = 3$.
4. The RL start from pole $s = 1$, $s = -2$, and $s = -3$.
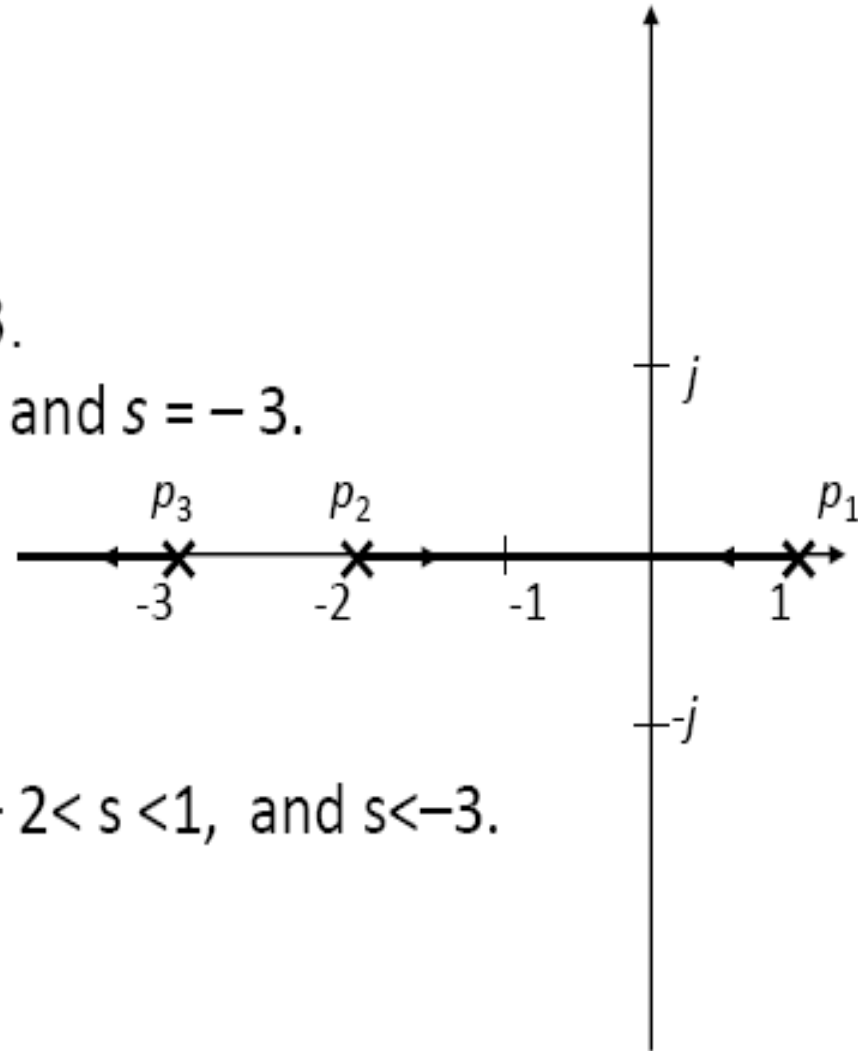
5. The RL occurs on the real axis for $-2 < s < 1$, and $s < -3$.

Because the number of roots is odd.

6. The real-axis asymptotes intercept is evaluated as;

$$\sigma_a = \frac{\sum poles - \sum zeros}{n - m}$$

$$\sigma_a = \frac{(1) + (-2) + (-3) - 0}{3 - 0} = \frac{-4}{3} = -1.33$$
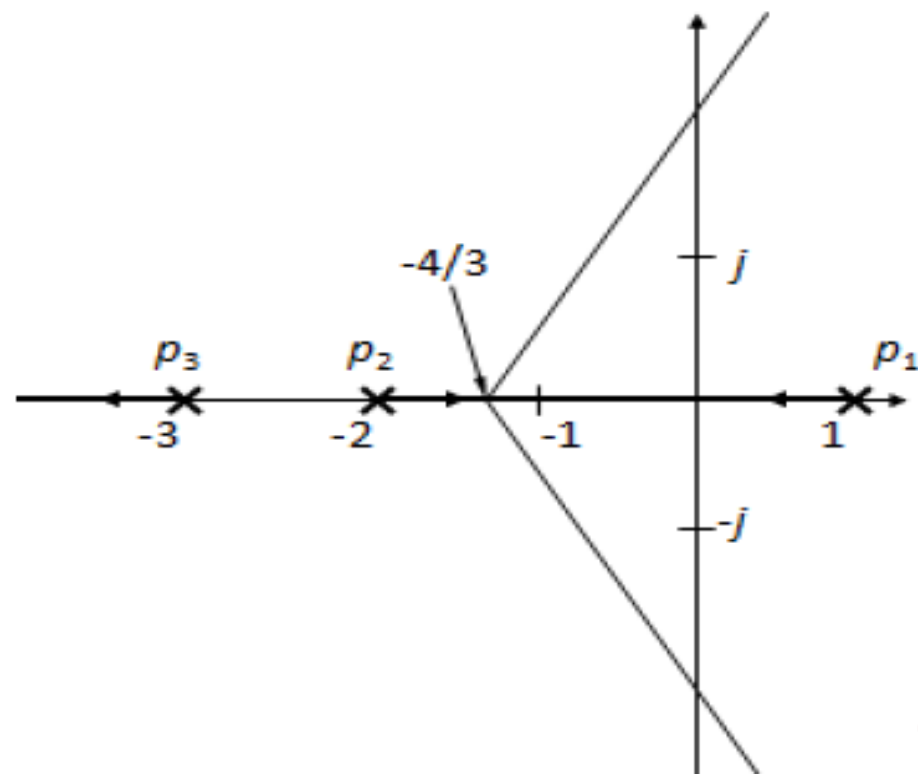
- The angles of the asymptotes that intersect at - 3, given by :

$$\theta_a = \frac{(2i+1)\pi}{n-m} = \frac{(2i+1)\pi}{3-0}$$

For $i = 0$, $\theta a = 60°$

For $i = 1$, $\theta a = 180°$

For $i = 2$, $\theta a = 300°$

7. The breakaway point, σ, can be determine as;

$$\boxed{\frac{dK}{ds} = 0 \quad or \quad \frac{dG(s)H(s)}{ds} = 0} \qquad G(s)H(s) = \frac{K}{(s-1)(s+2)(s+3)}$$

$$G(s)H(s) = \frac{K}{(s-1)(s+2)(s+3)} = \frac{K}{s^3 + 5s^2 + 6s - s^2 - 5s - 6}$$

$$\frac{-K(3s^2 + 10s + 6 - 2s - 5)}{(s^3 + 5s^2 + 6s - s^2 - 5s - 6)^2} = 0$$
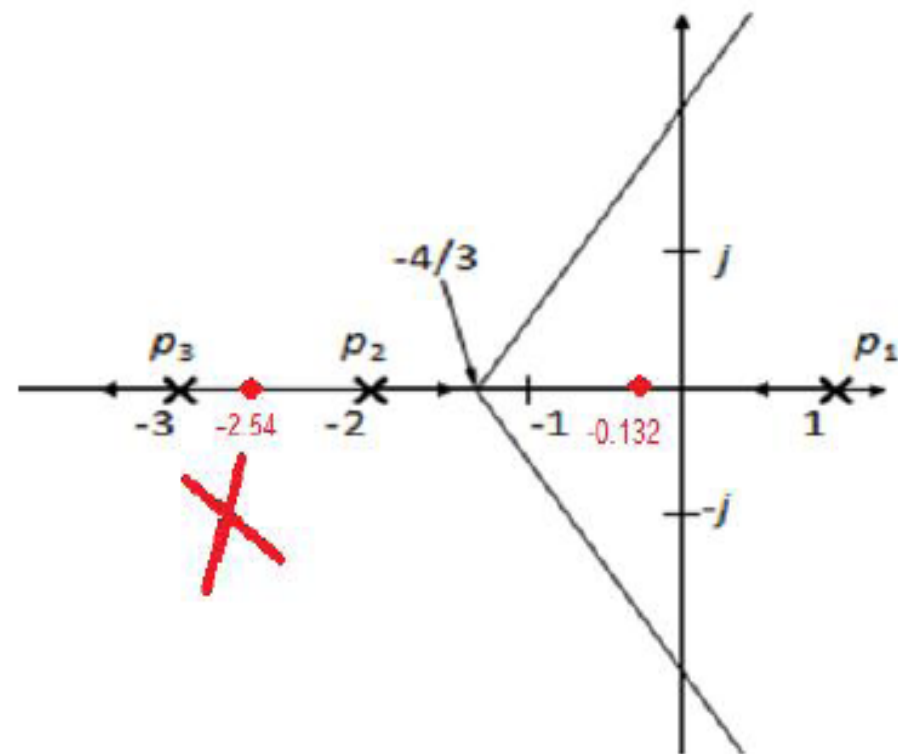
$$-K(3s^2 + 10s + 6 - 2s - 5) = 0$$

$$3s^2 + 10s + 6 - 2s - 5 = 0$$

$$3s^2 + 8s + 1 = 0$$

$$s_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-8 \pm \sqrt{64 - 4*3*1}}{2*3}$$

$$s_{1,2} = \frac{-8 \pm \sqrt{52}}{6} = \frac{-8 \pm 7.2}{6}$$

s_1 = -0.132 and s_2 = -2.54 (ignore s_2)

8. Find the points where the RL may cross the imaginary-axis.

The characteristic equation:

$$1 + G(s)H(s) = 1 + \frac{K}{(s-1)(s+2)(s+3)} = 0$$

$$(s-1)(s+2)(s+3) + K = 0$$

$$s^3 + 5s^2 + 6s - s^2 - 5s - 6 + K = 0$$

$$s^3 + 4s^2 + s + K - 6 = 0$$

$$s = j\omega$$

$$(j\omega)^3 + 4(j\omega)^2 + (j\omega) + K - 6 = 0$$

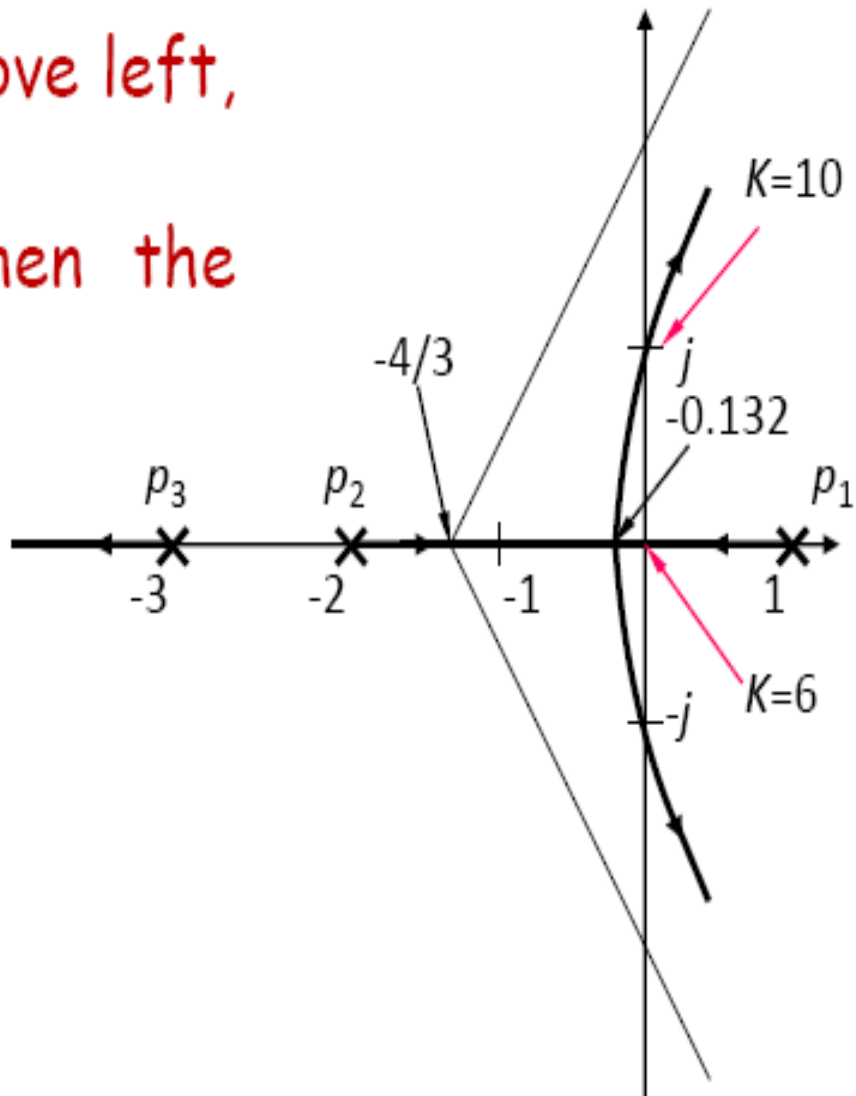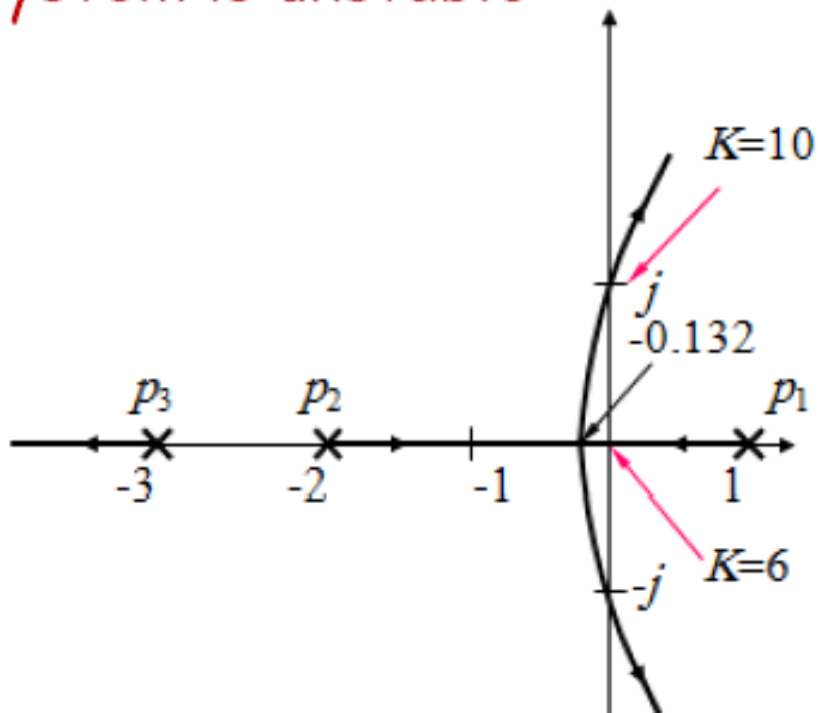$$-j\omega^3 - 4\omega^2 + j\omega + K - 6 = 0$$

$$-j\omega^3 + j\omega = 0$$

$$\omega^3 = \omega \qquad\qquad \omega = \pm 1$$

$$\omega = \pm 1$$

$$-4\omega^2 + K - 6 = 0 \quad at \quad \omega = \pm 1$$

$$-4 + K - 6 = 0 \quad Then \quad K = 10$$

➤ As K is increased p1 and p3 move left, but p2 move right.

➤ When K increase than 10 then the system is unstable

With Our Best Wishes

Automatic Control (1)

Course Staff

Thank You
For Your Attention

Mohamed Ahmed Ebrahim

Associate Prof. Dr . Mohamed Ahmed Ebrahim